Exploring the Rules in Species Counterpoint

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Abstract. In this short paper, we present a rule-based program for generating the upper part of the first and second species counterpoint. Using this program, we explore the music space/list of the possible answers given a cantus firmus. In future works, we hope to combine rule-based methods and machine learning techniques in algorithmic composition.

Keywords: Species counterpoint, music, rule-based, algorithmic composition

1 Introduction

Counterpoint is the study of writing melody lines harmonically and rhythmically. Learning how to write counterpoint is required in music theory courses since the counterpoint has been commonly used in classical music, especially in Baroque music. In 1725, Johann Joseph Fux, an Austrian composer, published a book, Gradus ad Parnassum, to teach how to write the counterpoint. For this pedagogical purpose, he developed five species counterpoint of increasing complexity.

In the species counterpoint, there is always a given melody (called cantus firmus) and students are supposed to write an accompaniment with several harmonic, melodic and rhythmic rules. Using this method of composing, the creative music writing process starts to resembles a logical/musical game and a dynamical system: to produce a good accompaniment melody, all students have to do is to combine the given information on the existing melody with certain rules.

Within each species, there are two types of rules: melodic and harmonic. Melodic rules are those govern the development of the melodic accompaniment line students are writing. For instance, there is a rule that two notes in the line should not be too far away in pitch. Harmonic rules are those govern the relationship between the cantus firmus and the counterpoint. For instance, there is a rule that all notes in the first species have to form consonance intervals. More details will be given in the next section.

Practically, one way to solve this puzzle systematically is to list all the harmonic candidates first, as shown in Figure 1, and then consider the melodic motion. To get the right melodic motions is essentially to find a path traversing the black notes with the constraint of the rules. Basically, we need to pick one note from the candidates in each bar and examine if there have been any
violations of the melodic rules. We implemented this process in the system to generate all possible answers. More details will be given in Section 3.

Since this research is still in its initial phase, we only consider writing the first and second species when the cantus firmus is at the lower part. In [1], the authors just explored the first species using machine learning techniques and got interesting results. In fact, there have been many efforts been made on using machine learning techniques [1–4] to generate species counterpoint. However, there have been less efforts in developing a rule-based system since the job is tedious and requires a lot of computational power. One such system is PRAENESTE [5]. It implemented more comprehensive rules than our system, but did not looked into the size of space of the possible answers.

Researches on counterpoint are important since the counterpoint has a fundamental role in classical music. Species counterpoint is particularly interesting because it reflexes the human efforts to summarise explicitly what rules are present in music, which are logical explanations on the human music creativity. Computers, on the other hand, excel at rules and logic but not yet as authentic in creating arts. Therefore, those rules are not only useful for music students, but also have the potential in directing machines to generate music more harmonically and rhythmically, and provide a platform for verifying the rules extracted by machine learning techniques. Eventually, we hope to integrate machine learning techniques with rule-based programming, two avenues for artificial intelligence, since, in human learning, we learn from experience and from explicit rules together all the time. Another purpose is to give clues on the magnitude of the music space when different rules are present. This would help us figure out the size of the data we need to perfectly deduce the rules in counterpoint using machines. More details will be given in Section 4 and Section 5.

The structure of the rest of the paper will be as follows: first we give explicitly the rules we used in generate the musical lines; second, the structure of our system is given; next, we show the sample results of the generated music and the size of the constrained music space and give analysis on these results; in the last section, we re-emphasise our conclusions and discuss future works to be done.

2 Rules of Counterpoint Species

There are many resources one could find about the counterpoint rules. We use the rules described in [6], which summarises the official book [7]. However, we
did find that there are rules from other sources which have more rules than our
sources. In addition, we did not implement the ‘soft rules’ in [7], which are only
recommended instead of mandated. To make the paper self-contained and also
show clearly which rules we actually implemented, in the following subsections,
we give the lists of the rules for all species, the rules for the first species, and
the rules for the second species.

2.1 Consideration for All Species

The following rules are implemented in both the first and second species gener-
ator:

- Harmonic: ‘The counterpoint must be in the same mode as the cantus firm-
us.’ Taken the interpretation from [6], practically, we open the piece with
a unison, octave, or perfect fifth interval between the cantus firmus and the
counterpoint. There is another rule that the opening should be an octave
or unison when the cantus firmus is in the upper part, but since we just
consider the case when the cantus firmus is in the lower part, this is not
included.
- Melodic: Augmented fourths/tritones, sevenths, any interval greater than
an octave, descending sixths whether major or minor, and ascending major
sixths are the skips which are forbidden in the melody of the counterpoint.
- Melodic: ‘Exposed tritones’, the run of notes in a single direction which
forms an augmented 4th from end to end, are forbidden in the melody of the
counterpoint.
- Melodic: Leaps of an ascending minor sixth or octave, or a descending octave
in the counterpoint melody must be ‘recovered’. Recovery here means that
such a leap must be followed by a step back into the range covered by the
leap.
- Melodic: Avoid repeated notes. Although in some cases this is not a ‘hard
rule’, we still implemented this to see the effects of this rule.

2.2 First Species

The rhythmic rule of the first species counterpoint is that there is only one
counterpoint note for each cantus firmus note. The rules govern the pitch features
are as follows:

- Harmonic: Dissonant intervals between the cantus firmus and the counter-
point are forbidden. Dissonant intervals include the minor second, major
second, augmented fourth, and sevenths.
- Melodic: Parallel fifths and octaves are forbidden. Parallel intervals are the
pattern that the interval formed by the counterpoint and the cantus firmus
is a fifth/octave at time $t$ and then again a fifth/octave at time $t + 1$.
- Harmonic: In the next to the last bar, there must be a major sixth if the
cantus firmus is in the lower part.
2.3 Second Species

The rhythmic rule of the second species counterpoint is that there are two counterpoint notes (including a possible rest at the beginning) correspond to each cantus firmus note except the last note, where it is the same with the first species. The rules govern the pitch features are as follows:

- Harmonic: The first note in each bar (the note on the strong beat) must be consonant with the cantus firmus.
- Harmonic: The second note in a bar (the note on the weak beat) may be dissonant, but only if it is approached and left by a step.
- Melodic: Parallel fifths and octaves are forbidden.
- Harmonic: In the last three notes, we use the formula of a fifth, a sixth, and an octave above the cantus firmus.

3 The System

The work-flow is shown in Figure 2. First, we create the cantus firmus using music scoring software like Musescore and Finale, then save the cantus firmus as ‘.xml’ format. Second, the data is loaded to our main program. As described in Section 1, we select all the harmonic candidates for the first note, the mid notes, and last notes, and then run the melodic rules to sieve out the candidates which do not satisfy the rules. At the same time, we also create a log file which records what were the rules which are violated. In the end of the process, we record all the melodies which satisfy the rules into a single file. Out of the melodies, we select one randomly, display and output a midi file of the melody.

We use the music21 package [8] in python to build this system. All the interval and rhythmical calculations are done using the package. Our main contribution is to write the functions which generate harmonic candidates and examine the melodic rules.
Our system is not the most efficient, but the code is logical and easy to read, and therefore is more extensible and has more flexibility. Users can add in and change the rules easily as they want. This is important since, as we mentioned before, the sources for species counterpoint rules are not always the equivalent with each other. We plan to optimise the program and put it on Github if there is interest.

4 Results

In this section, we show the sample results of the generated music and the size of the constrained music space and analysis these results.

4.1 First Species

First species counterpoint follows the rules described in Section 2. Figure 3 shows an example of a first species counterpoint generated by our system.

\[\text{Fig. 3. Generated first species example. The cantus firmus is at the bottom.}\]

4.2 Second Species

Second species counterpoint follows the rules described in Section 2. Figure 4 shows a example of a second species counterpoint generated by our system.

\[\text{Fig. 4. Generated second species example. The cantus firmus is at the bottom.}\]
4.3 Space Size Analysis

For first species counterpoint, because we have the one-to-one relationship between the cantus firmus and the counterpoint, the total number of answers of a five note long cantus firmus would be $12^5 = 248832$ (only consider the 12 semitones in an octave); for a four note long cantus firmus: $12^4 = 20736$; for a three note long cantus firmus: $12^3 = 1728$. We can see that the growing pattern is exponential. Similarly for the second species counterpoint: for a five note long cantus firmus, $12^9 = 5159780352$; for a four note long one, $12^7 = 35831808$, and for a three note long one, $12^5 = 248832$. It is also an exponential growth.

In Table 1, we give the number of answers of cantus firmus of different length after considering all the rules in Section 2. The actual cantus firmus are shown in Figure 5. The rests of the shorter length ones are not in the input data to generate the counterpoint; they are here for aesthetic reasons.

![Three cantus firmus of different length](image)

From Table 1, we can see that the rules are particularly effective when the length is small. This is intuitively right since the rules on the first note and the rules from the closing formula are very strict. We can also see that the exponential growth pattern is still there, but the space of constrained music is significantly smaller.

<table>
<thead>
<tr>
<th>species/length</th>
<th>note length = 3</th>
<th>note length = 4</th>
<th>note length = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>first species</td>
<td>12</td>
<td>60</td>
<td>143</td>
</tr>
<tr>
<td>second species</td>
<td>4</td>
<td>532</td>
<td>8030</td>
</tr>
</tbody>
</table>

Table 1. The size of the counterpoint answer space with varying length of cantus firmus
4.4 Constraints

In Table 2, we show the number of answers under the effect of different rules. The more eliminations there are, the more ‘powerful’ the rule is and the violations of the rule are more likely, at least in our special cases provided in Figure 5.

As expected, the rules regarding the first note and the last notes are the most powerful. The tritone rule is the weakest. The parallel fifth seems to eliminate more than the parallel octave rule. Forbidden skips, recovery and the step approach rules are very important ones since they are all hard-rules and very likely to happen. The repeat rule also eliminates lots of possibilities but since it is a soft rule, we need to decide on it case-by-case.

Table 2. The size of the counterpoint answer space with varying rule components

<table>
<thead>
<tr>
<th>rules/species</th>
<th>note length = 3</th>
<th>note length = 4</th>
<th>note length = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st species</td>
<td>2nd species</td>
<td>1st sp</td>
</tr>
<tr>
<td>first note</td>
<td>1296</td>
<td>165888</td>
<td>15552</td>
</tr>
<tr>
<td>forbidden skips</td>
<td>16</td>
<td>2</td>
<td>72</td>
</tr>
<tr>
<td>tritone run</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>recover</td>
<td>0</td>
<td>4</td>
<td>28</td>
</tr>
<tr>
<td>repeat</td>
<td>0</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>step approach</td>
<td>NA</td>
<td>6</td>
<td>NA</td>
</tr>
<tr>
<td>parallel fifth</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>parallel octaves</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>last notes</td>
<td>1296</td>
<td>248688</td>
<td>15552</td>
</tr>
</tbody>
</table>

For a music student, this table is suggesting that s/he should pay more attention to possible parallel fifth, forbidden skip, recovery and step approach in the second species. From another perspective, if one is to summarise the rules just from data, to get a reasonable confidence on the rules, one would need at least a large proportion of the numbers in the table. Similarly, for machines to learn, to perfectly learn the rules just from data, the magnitude of the data should be of similar magnitude to the numbers in the table. The mystery of how perception of acoustic dissonance and consonance helped Fux with summarising these rules are a yet unsolved one.

5 Conclusion and Future Works

In this project, we created a first and second species counterpoint answer generator. We provide the numbers of answers for sample cantus firmus of different length and analyse the numbers: how are they meaningful computationally and musically.

In future works, we would like to examine more samples and complete the third, fourth and fifth species counterpoint. Optimisation of the algorithms and
getting more computational power is also on the to-do list. Finally, our goal is to combine a machine learning system with the rule-based system and achieve an integrated intelligent system.

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References